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DIFFUSION OF TRANSIENT ELECTROMAGNETIC  
FIELDS THROUGH SATURATED FERROMAGNETIC  
MEDIA (U)

FINAL REPORT

by  
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Submitted by



CONTRACT DACA87-69-C-0009 MOD 2

Prepared for

U. S. ARMY ENGINEER DIVISION, HUNTSVILLE  
CORPS OF ENGINEERS

Approved for public release; distribution unlimited

JUNE 1970

## ABSTRACT

Intense transient electromagnetic pulse fields may drive a ferromagnetic shield into saturation, thus reducing the shielding effectiveness of that shield. Often shielding computations ignore saturation effects due to the complexity of the equations. This memorandum reduces the complexity of the calculations by combining the approaches of several authors. The ferromagnetic material characteristics are incorporated directly into the field equations, amenable to computer solution.

A math model of the material characteristics is presented. The magnetic permeability at a point in the ferromagnetic material is expressed as a function of  $H$ . The boundry conditions of the magnetic field equations in the time domain are developed. The numerical solution for the field emerging on the inside surface of an infinite sheet is accomplished by solving a differential equation subject to the boundry conditions. Certain approximations are indicated to simplify the calculations. The computer flow diagram is illustrated.

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## 1. INTRODUCTION

When intense electromagnetic fields, such as the transient field associated with the nuclear electromagnetic pulse (NEMP), drives a ferromagnetic shield into the saturated region, a degradation in shielding effectiveness occurs. In the past the time domain solutions for NEMP shielding computations have ignored saturation because of the increased complexity of the equations which must be solved.

A numerical solution, amenable to computer utilization, for pulse transmission through an infinite ferromagnetic sheet was derived by Merewether[1]. A classical finite difference technique was used to solve the appropriate nonlinear diffusion equation for the field distribution inside the ferromagnetic sheet. Although his solution treated nonhomogeneity, an isotropic material was assumed.

A far more simple solution was presented in a paper by Young [2] and later in another paper by Ferber and Young [3]. In that brief traveling wave solution, the diffusion of the fields into the ferromagnetic material is approximated via solution of a first order linear differential equation. On that basis, a simple expression for diffusion time for computing saturation punch through is obtained. That work is useful for rough estimates of the effect of saturation but is severely lacking in accuracy because of many of the approximations made. Namely, the material is assumed to be both homogeneous and isotropic, and the nonlinear effects are ignored. That is, the permeability is assumed to remain constant. While obviously in the real world we know that the permeability, at least for the isotropic case, can be expressed as a single valued function of the magnetic field intensity at a point in the ferromagnetic material, and is obtainable by curve fitting to the B versus H characteristics.

Some interesting numerical solutions for steady state problems have been presented based on a scalar magnetic potential [4] and a solution based on a vector magnetic potential [5]. However, examination of Maxwell's equations for the time transient case, we see that a scalar vector potential can not be defined for time transient problems. This is because definition of a scalar magnetic potential can not satisfy both the curl and the divergence equation when we look at the differential equation form of the field equations. However, definition of a vector magnetic potential is valid.

This memorandum essentially presents Merewether's results [1], however, incorporates some interesting material from references [4] and [6]. Namely, the technique of incorporating the ferromagnetic material characteristics directly into the field equations, and a numerical approximation to the differential equation which is amenable to computer solution. It is hoped in the future that a solution based on a magnetic vector potential can be developed for problems such as this.

## 2. MATHEMATICAL MODEL OF MATERIAL CHARACTERISTICS

The magnetic behavior at a point in a ferromagnetic material is conventionally described in terms of the B versus H characteristics where the anisotropic nature of ferromagnetic materials results in the familiar hysteresis or B-H loop. We are specifically treating magnetic field solutions for that class of ferromagnetic materials which can be represented as isotropic media. Thus the magnetic permeability is a scalar point function as opposed to a tensor function. This scalar point function is derivable from a single valued averaged approximation of a normal B-H characteristic.

The average B versus H characteristics for a typical annealed low carbon steel [4] is plotted in Figure 1. This characteristic must be expressed in mathematical form as a single-valued function of B versus H. In almost all cases there is considerable latitude in what one may consider as a satisfactory mathematical representation of an averaged B-H curve. For the material in Figure 1, an excellent curve fit is obtained with the following equations

$$B = 4000\mu_0 H = 50.2656 \times 10^{-4} H \quad 0 \leq H \leq 79.5775 \quad (2-1)$$

$$B = \frac{10^7}{4\pi} \mu_0 [1 - 0.6 \exp(-0.0083776(H-79.5775))] \quad 79.5775 \leq H \leq 1069.6067 \quad (2-2)$$

$$79.5775 \leq H \leq 1069.6067$$

and

$$B = 0.9998 + \mu_0 (H-1069.6067) \quad 1069.6067 \leq H \quad (2-3)$$

These expressions are based on rationalized MKS units where B is in webers per square meter and H is in ampere turns per meter.

The magnetic permeability at a point in the ferromagnetic material is expressible as a function of H by the following relation.

$$\mu = \frac{B(H)}{H} \quad (2-4)$$

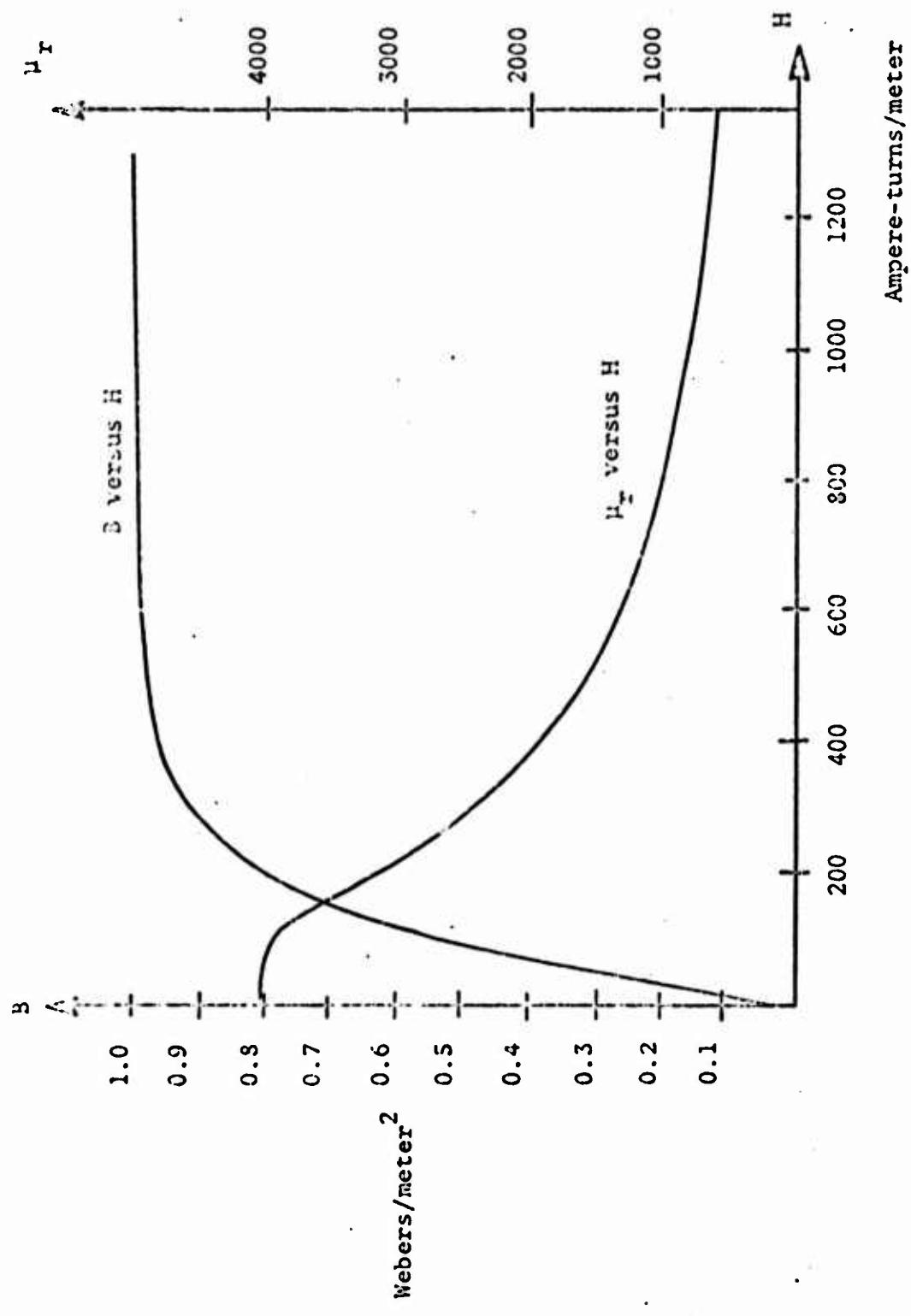


Figure 1.  $B$  versus  $H$  characteristics for an Annealed Low Carbon Steel  
(Relative permeability as a function of  $H$  is also illustrated)

By substitution of (2-1), (2-2), and (2-3) into (2-4), we obtain formulas for the permeability as single-valued functions of  $H$ . The curve showing relative permeability versus the magnetic field intensity  $H$  shown in Figure 1 was plotted based on these formulas and is presented to illustrate the strongly nonlinear behavior of the ferromagnetic material. For very strong saturating fields the relative permeability of the material approaches unity as we expect it to.

### 3. MAGNETIC FIELD EQUATIONS IN THE TIME-DOMAIN

The differential equations applicable to the fields diffusing through ferromagnetic material such as steel, operated in the saturated state, are obtainable from Maxwell's equations. Namely:

$$\nabla \times \bar{E} = - \frac{\partial \bar{B}}{\partial t} \quad (3-1)$$

$$\nabla \times \bar{H} = \sigma \bar{E} \quad (3-2)$$

and  $\nabla \cdot \bar{B} = \nabla \cdot \bar{D} = 0$  (3-3)

From (3-1) and (3-2) we can readily derive

$$\nabla^2 \bar{H} = \mu \sigma \frac{\partial \bar{H}}{\partial t} \quad (3-4)$$

If we consider the geometry illustrated in Figure 2, where we will investigate the propagation of a plane electromagnetic wave through a sheet of infinite extent and of thickness "a". To simplify the problem the wave is considered only to have a "y" component of magnetic field intensity and propagating in the positive "x" direction. On this basis, and by performing the curl operations indicated in equations (3-1) and (3-2) we obtain the relations

$$\frac{\partial H}{\partial x} = \sigma E \quad (3-5)$$

and  $\frac{\partial E}{\partial x} = \mu \frac{\partial H}{\partial t}$  (3-6)

Now, for a wave propagating into the sheet illustrated in Figure 2, the fields in Region I can be described by the following mathematical relation:

$$E(x,t) = E_1^+ (t - \frac{x}{c}) - E_1^- (t + \frac{x}{c}) \quad (3-7)$$

$$H(x,t) = \frac{E_1^+ (t - \frac{x}{c})}{\eta_0} + \frac{E_1^- (t + \frac{x}{c})}{\eta_0} \quad (3-8)$$

In region II, the electromagnetic field components are found via solution of equation (3-4). From this, and (3-5) it is readily found that the applicable differential equation reduces to:

$$\frac{\partial^2 H(x,t)}{\partial x^2} = \sigma\mu \frac{\partial H(x,y)}{\partial t} \quad (3-9)$$

Now the boundary conditions which must be satisfied between the three regions are obtainable from evaluating the equations for E and H in regions I & III at the surfaces. For instance, by evaluating (3-7) & (3-8) at  $x=0$  we obtain:

$$E(0,t) + \eta_0 H(0,t) = 2E_1^+(t) \quad (3-10)$$

Similarly, from the equations for E and H for region III (not presented) we obtain:

$$E(a,t) - \eta_0 H(a,t) = 0 \quad (3-11)$$

The problem at hand is the solution of the differential equation (3-9), subject to the boundary conditions (3-10) and (3-11). Utilizing the curl relations (3-5) and (3-6), the boundary condition equations become:

$$\frac{1}{\sigma} \frac{\partial}{\partial x} H(0,t) + \eta_0 H(0,t) = 2E_1^+(t) \quad (3-12)$$

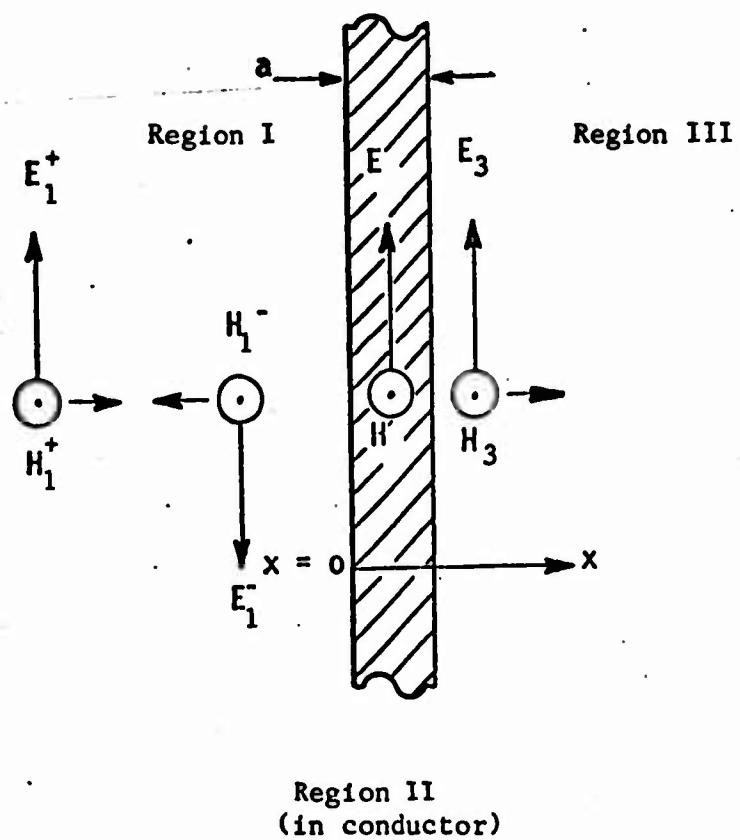


Figure 2. Plane Wave Propagation through an Infinite Sheet

$$\frac{1}{\sigma} \frac{\partial}{\partial x} H(a, t) - \eta_0 H(a, t) = 0 \quad (3-13)$$

The electric field transmitted through the sheet into region III, which is of prime interest, is given by (at  $x = a$ ):

$$E_3(t) = \eta_0 H(a, t) \quad (3-14)$$

We will proceed to indicate a solution for  $H(a, t)$  based on numerical methods.

#### 4. NUMERICAL SOLUTION AND COMPUTER FLOW DIAGRAM

To solve for the field emerging on the inside surface of the infinite sheet, it is necessary to solve the differential equation (3-9) subject to the boundary conditions (3-12) and (3-13). As a first step, a rectangular mesh of points is described in the x-t plane. In the x direction nodes are taken at uniform spacings between  $x = 0$  and  $x = a$ . In the t direction, nodes are taken at uniform spacing from  $t = 0$  to  $t = T_1$ , where  $T_1$  is the maximum estimated duration response time. The derivations in (3-9), (3-12) and (3-13) are then replaced by the difference approximations:

$$\frac{H(x+1,t) + H(x-1,t) - 2H(x,t)}{\Delta_x^2} = \sigma\mu \frac{H(x,t+1) - H(x,t-1)}{2\Delta_t} \quad (4-1)$$

$$2E_1^+(t) = \frac{1}{\sigma} \frac{H(+1,t) - 2H_1^+(t)}{2\Delta_x} + \eta_0 H(0,t) \quad (4-2)$$

$$0 = \frac{1}{\sigma} \frac{H(a,t) - H(a-1,t)}{\Delta_x} - \eta_0 H(a,t) \quad (4-3)$$

Where in the previous equations the permeability is taken as the mathematical model of the form indicated in Section 2.0 of this memorandum, and is evaluated at each point in the mesh once each iteration cycle. The solution is effected by rearranging (4-1) to solve for  $H(x,t)$ , and by proceeding sequentially through the array through sufficient iterations until the solution has converged to reasonable accuracy. The subject of convergence will not be discussed here, but in computer programs investigated in [4] and [6] it was found that sufficient accuracy is obtained with well under 100 iteration cycles. Accuracy will be related to mesh coarseness, magnitudes of the driving field, the material permeability math model, and other factors.

A computer flow diagram is indicated in Figure 3. By computer execution, accurate time domain solutions for the wave emerging on the far side of the metal sheet can be obtained.

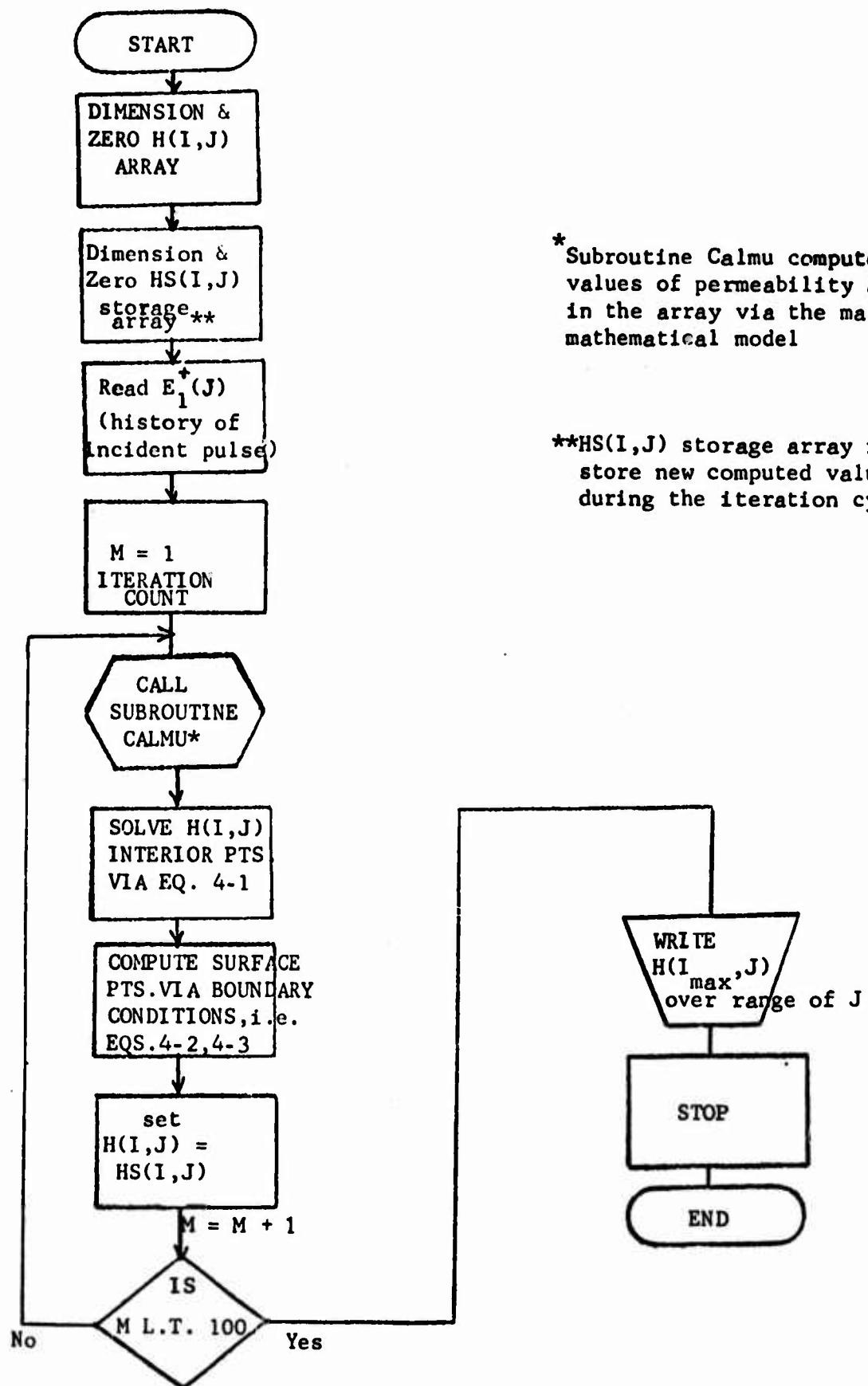


Figure 3. COMPUTER PROGRAM FLOW DIAGRAM FOR SOLUTION OF DIFFUSION OF ELECTROMAGNETIC PULSE THROUGH FERROMAGNETIC MATERIAL.

## 5.0 CONCLUSIONS

When pulse amplitude of an incident electromagnetic wave drives the surface of a shield into saturation, degradation in shielding effectiveness occurs. The method described in this memorandum permits accurate computer solution for the time response of the wave propagating through a ferromagnetic sheet.

## REFERENCES

- [1] "Electromagnetic Pulse Transmission through a Thin Sheet of Saturable Ferromagnetic Material of Infinite Surface Area", D. E. Merewether, I.E.E.E. Transactions on Electromagnetic Compatibility, Volume EMC-11, No. 4, November 1969.
- [2] "Ferromagnetic Shielding Related to the Physical Properties of Iron", F. J. Young, paper presented at I.E.E.E. Symposium on Electromagnetic Compatibility, Seattle, Washington, July 1968.
- [3] "Shielding Electromagnetic Pulses by Use of Magnetic Materials", R. R. Ferber and F. J. Young, I.E.E.E. Symposium on Electromagnetic Compatibility, Atlanta, Georgia, October 1969.
- [4] "The Three-Dimensional Nonlinear Magnetic Field Boundary Value Problem and Its Numerical Solution", D. J. Kozakoff and F. O. Simons, Jr., I.E.E.E. Transactions on Magnetics, December 1969.
- [5] "Nonlinear Magnetic Field Solutions based on a Vector Magnetic Potential", F. O. Simons, Jr., D. J. Kozakoff, and R. C. Harden, paper presented at I.E.E.E. International Conference on Magnetics (INTERMAG), Washington, D. C., April 1970.
- [6] "Magnetic Field Solutions for One Class of Nonlinear Boundary Value Problems", F. O. Simons, unpublished doctoral dissertation, University of Florida, June 1965.

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R & D

(Revert to classification of report. Code of authors and funding annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate Author) <b>Sperry Rand Corporation Space Support Division Huntsville, Alabama</b>		2a. REPORT SECURITY CLASSIFICATION <b>Unclassified</b>	
3. REPORT TITLE <b>Diffusion of Transient Electromagnetic Fields through Saturated Ferromagnetic Media.</b>		3a. GROUP <b>None</b>	
4. DESCRIPTIVE NOTES (Type of report and Inclusive dates) <b>Final (Inclusive dates N/A)</b>			
5. AUTHOR(S) (First name, middle initial, last name) <b>D. J. Kozakoff</b>			
6. REPORT DATE <b>June 1970</b>		7a. TOTAL NO. OF PAGES <b>19</b>	7b. NO. OF REPS <b>6</b>
8a. CONTRACT OR GRANT NO. <b>DACA87-69-C-0009 Mod 2</b>		8a. ORIGINATOR'S REPORT NUMBER(S) <b>SR-48114-D</b>	
8b. PROJECT NO. <b>None assigned</b>		8b. OTHER REPORT NUMBER(S) (Any other numbers that may be assigned this report) <b>None as of 13 April 1971</b>	
10. DISTRIBUTION STATEMENT <b>Approved for public release, distribution unlimited.</b>			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY <b>U.S. Army Engineer Division, P.O. Box 1600 West Station Huntsville, Alabama 35807</b>	
13. ABSTRACT <p>Intense transient electromagnetic pulse fields may drive a ferromagnetic shield into saturation, thus reducing the shielding effectiveness of that shield. Often shielding computations ignore saturation effects due to the complexity of the equations. This memorandum reduces the complexity of the calculations by combining the approaches of several authors. The ferromagnetic material characteristics are incorporated directly into the field equations, amenable to computer solution.</p> <p>A math model of the material characteristics is presented. The magnetic permeability at a point in the ferromagnetic material is expressed as a function of H. The boundary conditions of the magnetic field equations in the time domain are developed. The numerical solution for the field emerging on the inside surface of an infinite sheet is accomplished by solving a differential equation subject to the boundary conditions. Certain approximations are indicated to simplify the calculations. The computer flow diagram is illustrated.</p>			

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Unclassified  
Security Classification

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REF ID: REV WORDS	LINE A		LINE B		LINE C	
	ROLE	RT	ROLE	RT	ROLE	RT
Shielding Effectiveness						
Ferromagnetic Shield						
Electromagnetic Pulse Fields						
Nuclear Electromagnetic Pulse						
Scalar Magnetic Potential						
Vector Magnetic Potential						
Magnetic Field Equations (Time-Domain)						
H-Field Saturation Effects						

**Unclassified**

Security Classification